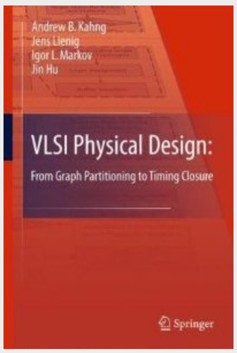
VLSI Physical Design: From Graph Partitioning to Timing Closure

Chapter 4 – Global and Detailed Placement



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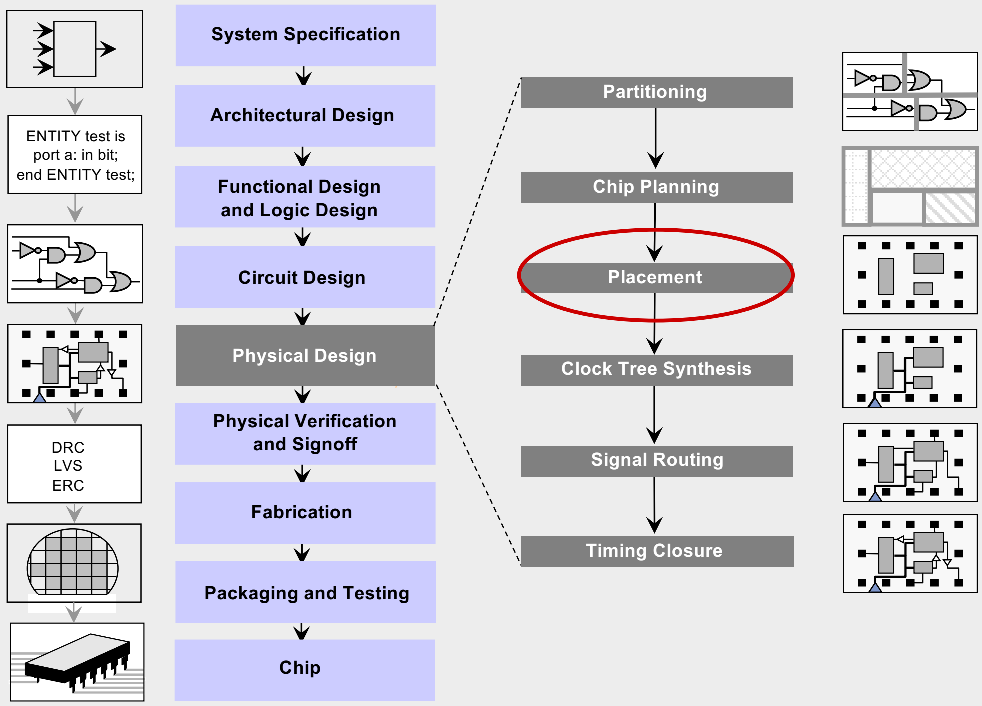
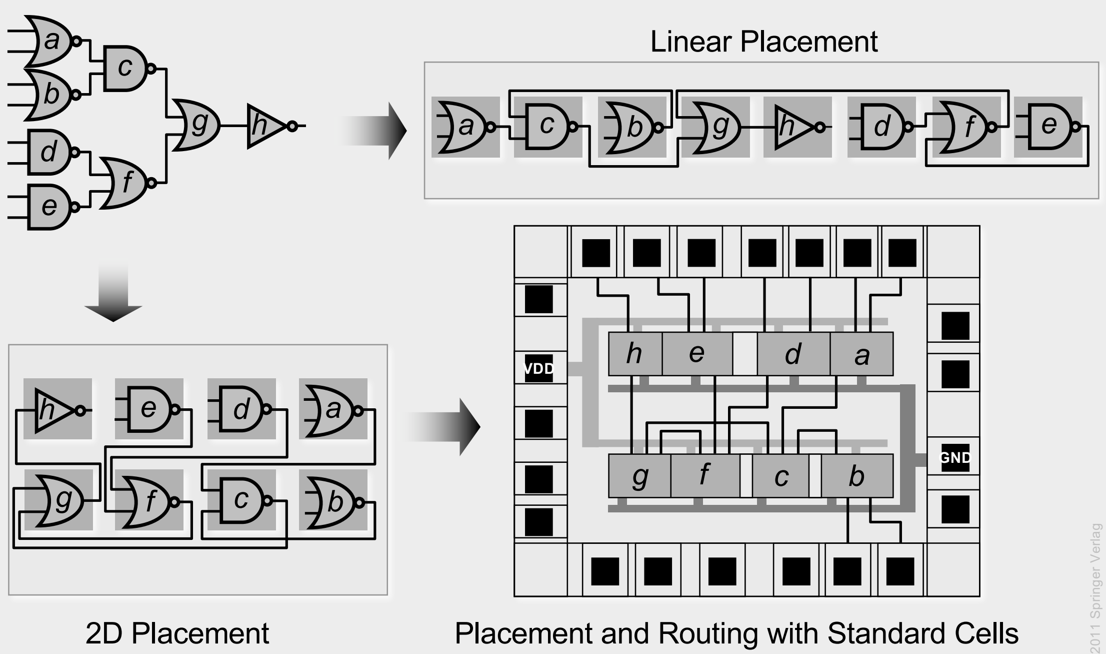
Chapter 4 – Global and Detailed Placement

4.1 Introduction



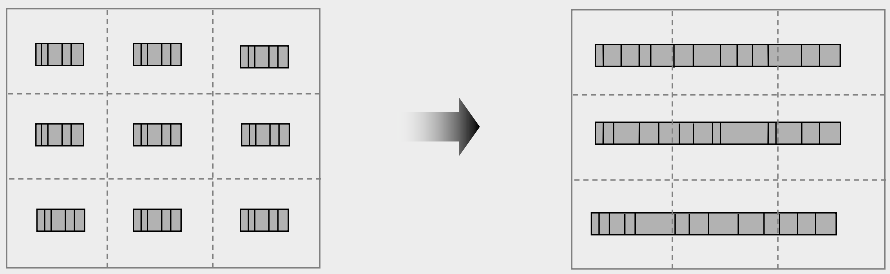
4.1 Introduction



4.1 Introduction

Global Placement

Detailed Placement



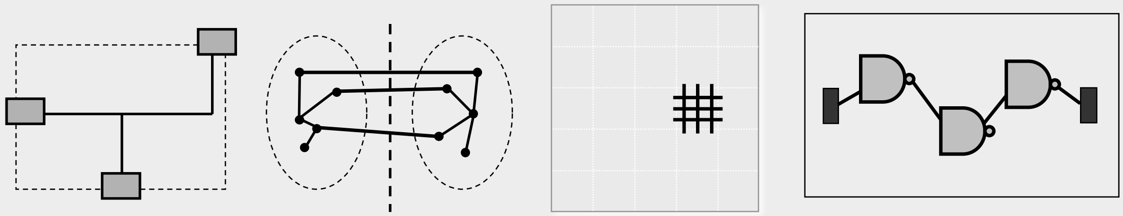
4.2 Optimization Objectives

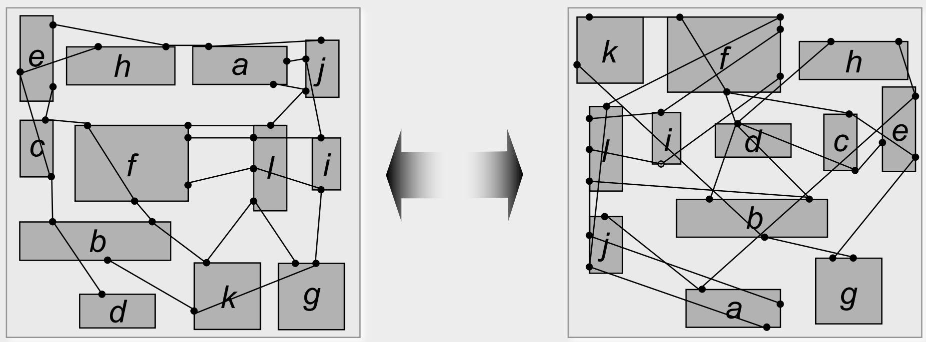
Total Wirelength

Number of Cut Nets

Wire Congestion

Signal Delay





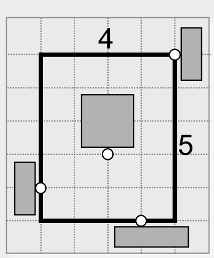
Wirelength estimation for a given placement

Half-perimeter wirelength (HPWL)

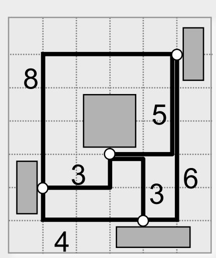
Complete graph (clique)

Monotone chain

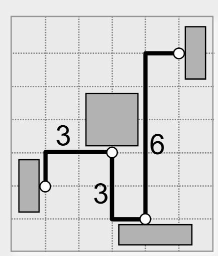
Star model



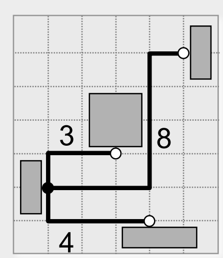
HPWL = 9



Clique Length = $(2/p)\Sigma_{e \in clique} d_M(e)$ = 14.5



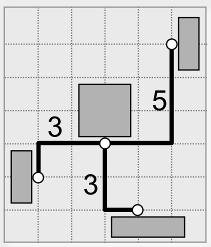
Chain Length = 12



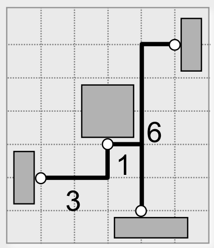
Star Length = 15

Wirelength estimation for a given placement (cont'd.)

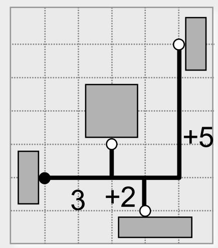
Rectilinear minimum spanning tree (RMST)	Rectilinear Steiner minimum tree (RSMT)	Rectilinear Steiner arborescence model (RSA)	Single-trunk Steiner tree (STST)



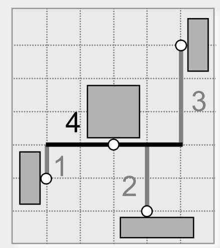
RMST Length = 11



RSMT Length = 10



RSA Length = 10

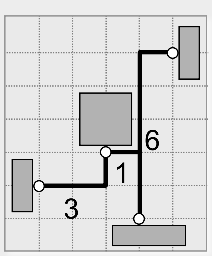


STST Length = 10

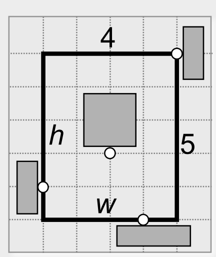
Wirelength estimation for a given placement (cont'd.)

Preferred method: Half-perimeter wirelength (HPWL)

- Fast (order of magnitude faster than RSMT)
- Equal to length of RSMT for 2- and 3-pin nets
- Margin of error for real circuits approx. 8% [Chu, ICCAD 04]



RSMT Length = 10



HPWL = 9

$$L_{\mathrm{HPWL}} = w + h$$

4.2 Optimization Objectives – Total Wirelength

Total wirelength with net weights (weighted wirelength)

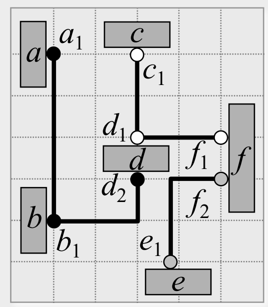
- For a placement P, an estimate of total weighted wirelength is

$$L(P) = \sum_{net \in P} w(net) \cdot L(net)$$

where $w(net)$ is the weight of net, and $L(net)$ is the estimated wirelength of net.

- Example:

Nets	Weights
$N_1 = (a_1, b_1, d_2)$	$w(N_1) = 2$
$N_2 = (c_1, d_1, f_1)$	$w(N_2) = 4$
$N_3 = (e_1, f_2)$	$w(N_3) = 1$



$$L(P) = \sum_{net \in P} w(net) \cdot L(net) = 2 \cdot 7 + 4 \cdot 4 + 1 \cdot 3 = 33$$

4.2 Optimization Objectives – Number of Cut Nets

Cut sizes of a placement

- To improve total wirelength of a placement P, separately calculate the number of crossings of global vertical and horizontal cutlines, and minimize

$$L(P) = \sum_{v \in V_P} \psi_P(v) + \sum_{h \in H_P} \psi_P(h)$$

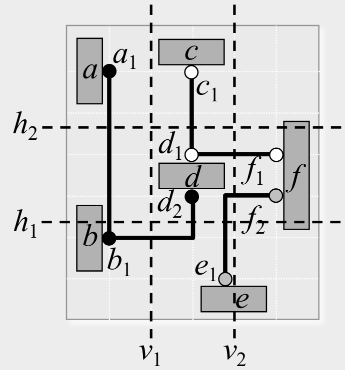
where $\Psi_P(cut)$ be the set of nets cut by a cutline *cut*

4.2 Optimization Objectives – Number of Cut Nets

Cut sizes of a placement

- Example:

  Nets
  $N_1 = (a_1, b_1, d_2)$
  $N_2 = (c_1, d_1, f_1)$
  $N_3 = (e_1, f_2)$

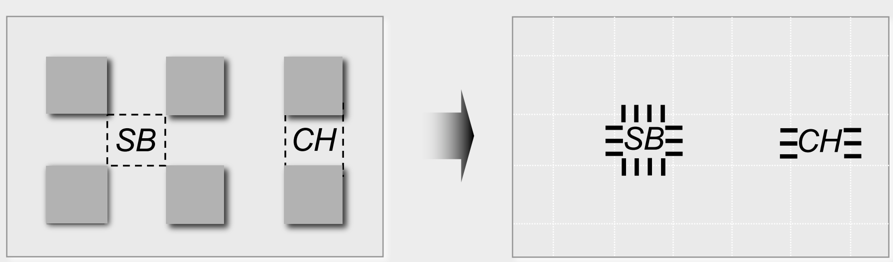


- Cut values for each global cutline
  $\psi_P(v_1) = 1$ $\psi_P(v_2) = 2$
  $\psi_P(h_1) = 3$ $\psi_P(h_2) = 2$
- Total number of crossings in P
  $\psi_P(v_1) + \psi_P(v_2) + \psi_P(h_1) + \psi_P(h_2) = 1 + 2 + 3 + 2 = 8$
- Cut sizes
  $X(P) = \max(\psi_P(v_1),\psi_P(v_2)) = \max(1,2) = 2$
  $Y(P) = \max(\psi_P(h_1),\psi_P(h_2)) = \max(3,2) = 3$

4.2 Optimization Objectives – Wire Congestion

Routing congestion of a placement

- Ratio of demand for routing tracks to the supply of available routing tracks
- Estimated by the number of nets that pass through the boundaries of individual routing regions



Wire capacities

4.2 Optimization Objectives – Wire Congestion

Routing congestion of a placement

- Formally, the local wire density $\varphi_P(e)$ of an edge e between two neighboring grid cells is

$$\varphi_P(e) = \frac{\eta_P(e)}{\sigma_P(e)}$$

where $\eta_P(e)$ is the estimated number of nets that cross e and
$\sigma_P(e)$ is the maximum number of nets that can cross e

- If $\varphi_P(e) > 1$, then too many nets are estimated to cross e, making P more likely to be unroutable.
- The wire density of P is $$\Phi(P) = \max_{e \in E}(\varphi_P(e))$$

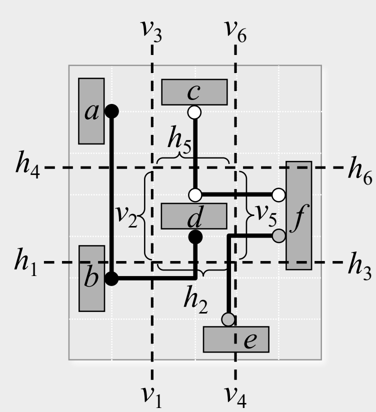
where E is the set of all edges

- If $\Phi(P) \leq 1$, then the design is estimated to be fully routable, otherwise routing will need to detour some nets through less-congested edges

4.2 Optimization Objectives – Wire Congestion

Wire Density of a placement

$\eta_P(h_1) = 1$ $\eta_P(v_1) = 1$
$\eta_P(h_2) = 2$ $\eta_P(v_2) = 0$
$\eta_P(h_3) = 0$ $\eta_P(v_3) = 0$
$\eta_P(h_4) = 1$ $\eta_P(v_4) = 0$
$\eta_P(h_5) = 1$ $\eta_P(v_5) = 2$
$\eta_P(h_6) = 0$ $\eta_P(v_6) = 0$



Maximum: $\eta_P(e) = 2$

$$\Phi(P) = \frac{\eta_P(e)}{\sigma_P(e)} = \frac{2}{3}$$ → Routable

4.2 Optimization Objectives – Signal Delay

Circuit timing of a placement

- Static timing analysis using actual arrival time (*AAT*) and required arrival time (*RAT*)
  - *AAT*(*v*) represents the latest transition time at a given node *v* measured from the beginning of the clock cycle
  - *RAT*(*v*) represents the time by which the latest transition at *v* must complete in order for the circuit to operate correctly within a given clock cycle.
- For correct operation of the chip with respect to setup (maximum path delay) constraints, it is required that $AAT(v) \leq RAT(v)$.

Global Placement

Global Placement

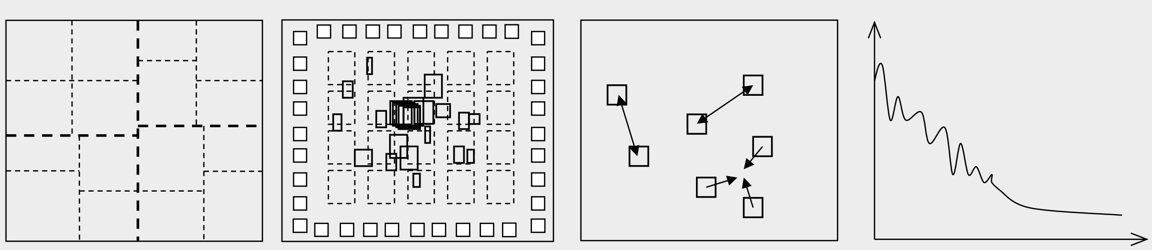
- Partitioning-based algorithms:
  - The netlist and the layout are divided into smaller sub-netlists and sub-regions, respectively
  - Process is repeated until each sub-netlist and sub-region is small enough to be handled optimally
  - Detailed placement often performed by optimal solvers, facilitating a natural transition from global placement to detailed placement
  - Example: min-cut placement
- Analytic techniques:
  - Model the placement problem using an objective (cost) function, which can be optimized via numerical analysis
  - Examples: quadratic placement and force-directed placement
- Stochastic algorithms:
  - Randomized moves that allow hill-climbing are used to optimize the cost function
  - Example: simulated annealing

Global Placement

Partitioning-based → Min-cut placement

Analytic → Quadratic placement; Force-directed placement

Stochastic → Simulated annealing

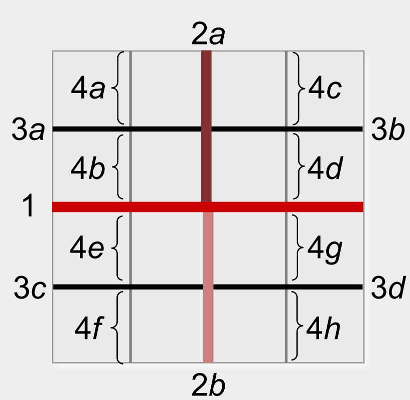


4.3.1 Min-Cut Placement

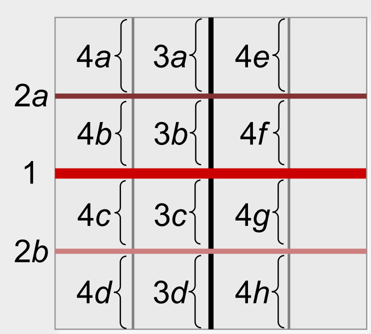
- Uses partitioning algorithms to divide (1) the netlist and (2) the layout region into smaller sub-netlists and sub-regions
- Conceptually, each sub-region is assigned a portion of the original netlist
- Each cut heuristically minimizes the number of cut nets using, for example,
  - Kernighan-Lin (KL) algorithm
  - Fiduccia-Mattheyses (FM) algorithm

4.3.1 Min-Cut Placement

Alternating cutline directions



Repeating cutline directions



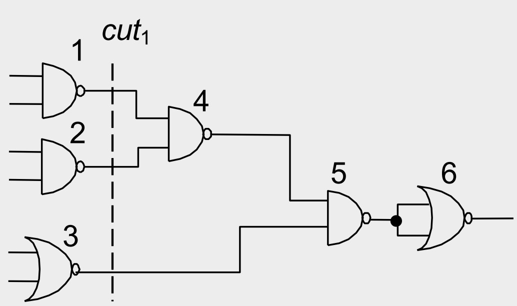
4.3.1 Min-Cut Placement

Input: netlist *Netlist*, layout area *LA*, minimum number of cells per region *cells_min*
Output: placement *P*

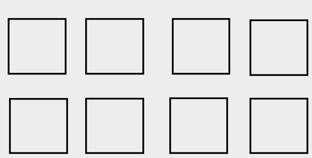
```
P = Ø
regions = ASSIGN(Netlist,LA)                   // assign netlist to layout area
while (regions != Ø)                           // while regions still not placed
   region = FIRST_ELEMENT(regions)             // first element in regions
   REMOVE(regions, region)                     // remove first element of regions
   if (region contains more than cell_min cells)
      (sr1,sr2) = BISECT(region)               // divide region into two subregions
                                               //   sr1 and sr2, obtaining the sub-
                                               //   netlists and sub-areas
      ADD_TO_END(regions,sr1)                  // add sr1 to the end of regions
      ADD_TO_END(regions,sr2)                  // add sr2 to the end of regions
   else
     PLACE(region)                             // place region
     ADD(P,region)                             // add region to P
```

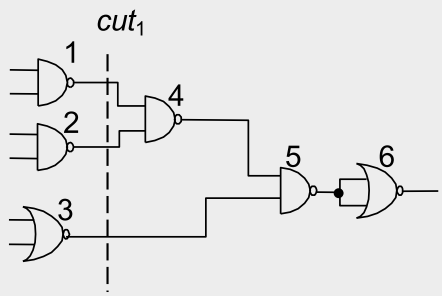

4.3.1 Min-Cut Placement – Example

Given:

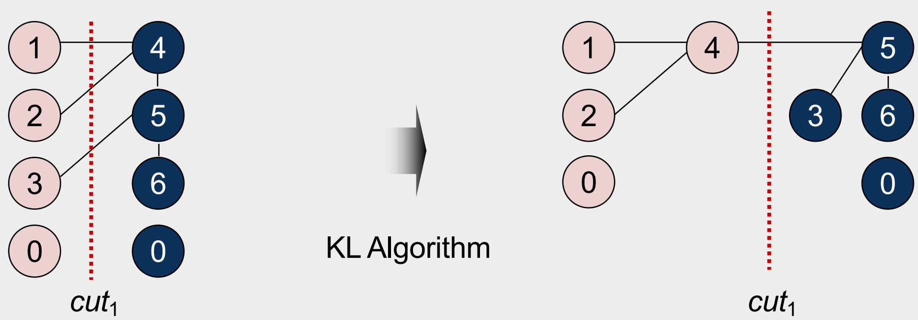


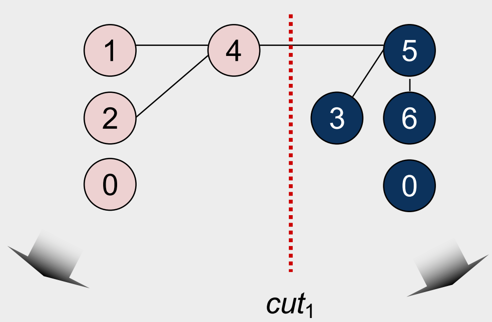
Task: 4 x 2 placement with minimum wirelength using alternative cutline directions and the KL algorithm





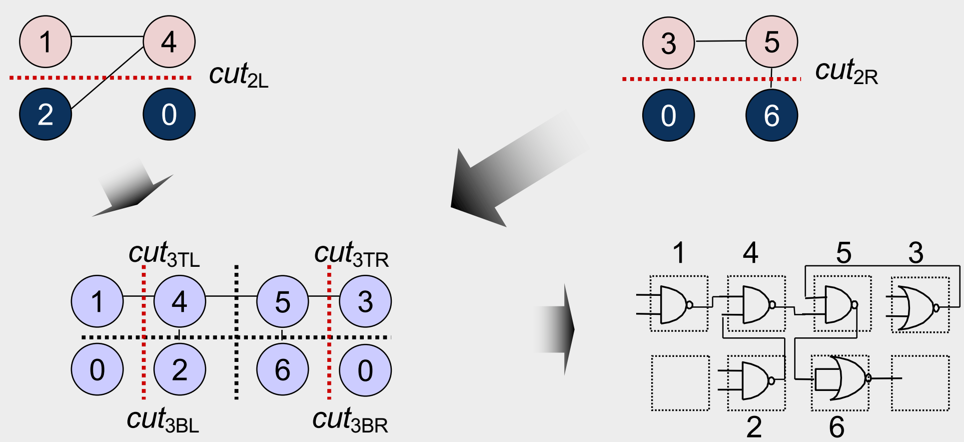
Vertical cut cut_1: L={1,2,3}, R={4,5,6}



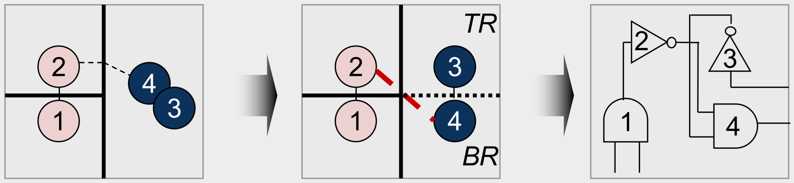


Horizontal cut cut_{2L}: T={1,4}, B={2,0}

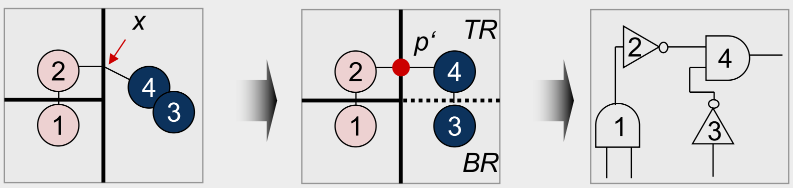
Horizontal cut cut_{2R}: T={3,5}, B={6,0}



4.3.1 Min-Cut Placement – Terminal Propagation



- Terminal Propagation
  - External connections are represented by artificial connection points on the cutline
  - Dummy nodes in hypergraphs



4.3.1 Min-Cut Placement

- Advantages:
  - Reasonably fast
  - Objective function can be adjusted, e.g., to perform timing-driven placement
  - Hierarchical strategy applicable to large circuits
- Disadvantages:
  - Randomized, chaotic algorithms – small changes in input lead to large changes in output
  - Optimizing one cutline at a time may result in routing congestion elsewhere

4.3.2 Analytic Placement – Quadratic Placement

- Objective function is quadratic; sum of (weighted) squared Euclidean distance represents placement objective function

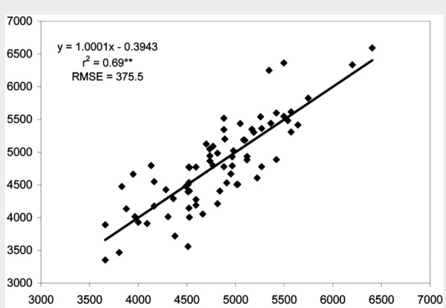
$$L(P) = \frac{1}{2} \sum_{i,j=1}^{n} c_{ij} \left(\left(x_i - x_j \right)^2 + \left(y_i - y_j \right)^2 \right)$$

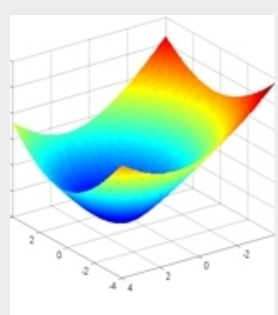
where n is the total number of cells, and $c(i,j)$ is the connection cost between cells i and j.

- Only two-point-connections
- Minimize objective function by equating its derivative to zero which reduces to solving a system of linear equations

4.3.2 Analytic Placement – Quadratic Placement

- Similar to Least-Mean-Square Method (root mean square)
- Build error function with analytic form: $E(a,b) = \sum (a \cdot x_i + b - y_i)^2$





4.3.2 Analytic Placement – Quadratic Placement

$$L(P) = \frac{1}{2} \sum_{i,j=1}^{n} c_{ij} \left((x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

where *n* is the total number of cells, and *c*(*i*,*j*) is the connection cost between cells *i* and *j*.

- Each dimension can be considered independently:

$$L_x(P) = \sum_{i=1, j=1}^{n} c(i,j)(x_i - x_j)^2 \qquad L_y(P) = \sum_{i=1, j=1}^{n} c(i,j)(y_i - y_j)^2$$

- Convex quadratic optimization problem: any local minimum solution is also a global minimum
- Optimal *x*- and *y*-coordinates can be found by setting the partial derivatives of $L_x(P)$ and $L_y(P)$ to zero

4.3.2 Analytic Placement – Quadratic Placement

$$L(P) = \frac{1}{2} \sum_{i,j=1}^{n} c_{ij} \left((x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

where *n* is the total number of cells, and *c*(*i*,*j*) is the connection cost between cells *i* and *j*.

- Each dimension can be considered independently:

$$L_x(P) = \sum_{i=1, j=1}^{n} c(i,j)(x_i - x_j)^2 \qquad L_y(P) = \sum_{i=1, j=1}^{n} c(i,j)(y_i - y_j)^2$$

$$\frac{\partial L_x(P)}{\partial X} = AX - b_x = 0 \qquad \frac{\partial L_y(P)}{\partial Y} = AY - b_y = 0$$

where *A* is a matrix with $A[i][j] = -c(i,j)$ when $i \neq j$,
and $A[i][i]$ = the sum of incident connection weights of cell *i*.

X is a vector of all the *x*-coordinates of the non-fixed cells, and b_x is a vector with $b_x[i]$ = the sum of *x*-coordinates of all fixed cells attached to *i*.

Y is a vector of all the *y*-coordinates of the non-fixed cells, and b_y is a vector with $b_y[i]$ = the sum of *y*-coordinates of all fixed cells attached to *i*.

4.3.2 Analytic Placement – Quadratic Placement

$$L(P) = \frac{1}{2} \sum_{i,j=1}^{n} c_{ij} \left((x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

where n is the total number of cells, and $c(i,j)$ is the connection cost between cells i and j.

- Each dimension can be considered independently:

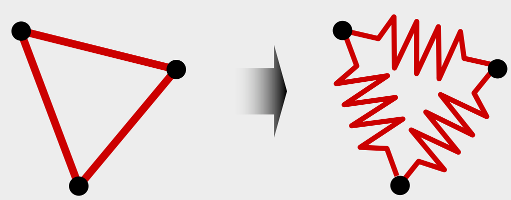
$$L_x(P) = \sum_{i=1,j=1}^{n} c(i,j)(x_i - x_j)^2 \qquad L_y(P) = \sum_{i=1,j=1}^{n} c(i,j)(y_i - y_j)^2$$

$$\frac{\partial L_x(P)}{\partial X} = AX - b_x = 0 \qquad \frac{\partial L_y(P)}{\partial Y} = AY - b_y = 0$$

- System of linear equations for which iterative numerical methods can be used to find a solution

4.3.2 Analytic Placement – Quadratic Placement

- Mechanical analogy: mass-spring system

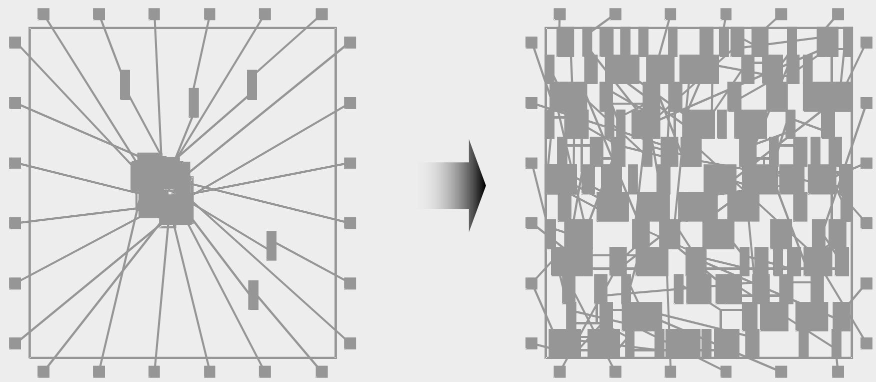


  – Squared Euclidean distance is proportional to the energy of a spring between these points
  – Quadratic objective function represents total energy of the spring system; for each movable object, the *x* (*y*) partial derivative represents the total force acting on that object
  – Setting the forces of the nets to zero, an equilibrium state is mathematically modeled that is characterized by zero forces acting on each movable object
  – At the end, all springs are in a force equilibrium with a minimal total spring energy; this equilibrium represents the minimal sum of squared wirelength

→ Result: many cell overlaps

4.3.2 Analytic Placement – Quadratic Placement

- Second stage of quadratic placers: cells are spread out to remove overlaps
- Methods:
  - Adding fake nets that pull cells away from dense regions toward anchors
  - Geometric sorting and scaling
  - Repulsion forces, etc.

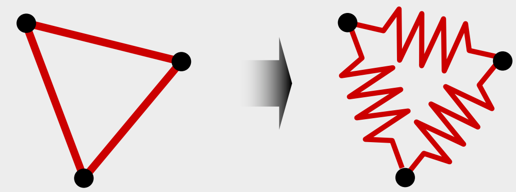


4.3.2 Analytic Placement – Quadratic Placement

- Advantages:
  - Captures the placement problem concisely in mathematical terms
  - Leverages efficient algorithms from numerical analysis and available software
  - Can be applied to large circuits without netlist clustering (flat)
  - Stability: small changes in the input do not lead to large changes in the output
- Disadvantages:
  - Connections to fixed objects are necessary: I/O pads, pins of fixed macros, etc.

4.3.2 Analytic Placement – Force-directed Placement

- Cells and wires are modeled using the mechanical analogy of a mass-spring system, i.e., masses connected to Hooke's-Law springs



- Attraction force between cells is directly proportional to their distance
- Cells will eventually settle in a force equilibrium → minimized wirelength

4.3.2 Analytic Placement – Force-directed Placement

- Given two connected cells *a* and *b*, the attraction force $\overrightarrow{F_{ab}}$ exerted on *a* by *b* is

  $$\overrightarrow{F_{ab}} = c(a,b) \cdot (\vec{b} - \vec{a})$$

  where
  - *c*(*a*,*b*) is the connection weight (priority) between cells *a* and *b*, and
  - $(\vec{b} - \vec{a})$ is the vector difference of the positions of *a* and *b* in the Euclidean plane

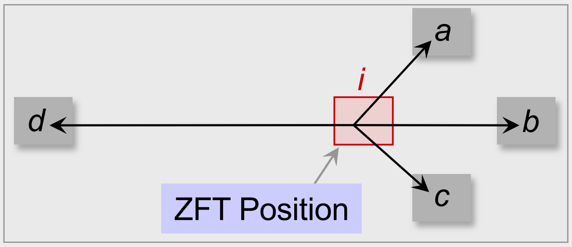
- The sum of forces exerted on a cell *i* connected to other cells 1… *j* is

  $$\overrightarrow{F_i} = \sum_{c(i,j) \neq 0} \overrightarrow{F_{ij}}$$

- Zero-force target (ZFT): position that minimizes this sum of forces

4.3.2 Analytic Placement – Force-directed Placement

Zero-Force-Target (ZFT) position of cell *i*



$$\min \vec{F_i} = c(i,a)\cdot(\vec{a}-\vec{i}) + c(i,b)\cdot(\vec{b}-\vec{i}) + c(i,c)\cdot(\vec{c}-\vec{i}) + c(i,d)\cdot(\vec{d}-\vec{i})$$

4.3.2 Analytic Placement – Force-directed Placement

Basic force-directed placement

- Iteratively moves all cells to their respective ZFT positions
- *x*- and *y*-direction forces are set to zero:

$$\sum_{c(i,j)\neq 0} c(i,j)\cdot(x_j^0 - x_i^0) = 0 \qquad \sum_{c(i,j)\neq 0} c(i,j)\cdot(y_j^0 - y_i^0) = 0$$

- Rearranging the variables to solve for x_i^0 and y_i^0 yields

$$x_i^0 = \frac{\sum_{c(i,j)\neq 0} c(i,j)\cdot x_j^0}{\sum_{c(i,j)\neq 0} c(i,j)} \qquad y_i^0 = \frac{\sum_{c(i,j)\neq 0} c(i,j)\cdot y_j^0}{\sum_{c(i,j)\neq 0} c(i,j)}$$

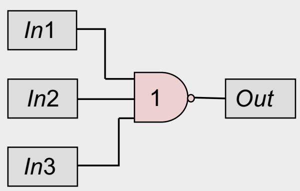
Computation of ZFT position of cell *i* connected with cells 1 … *j*

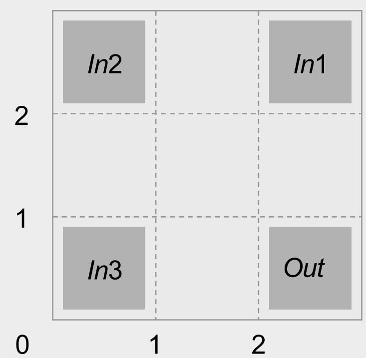
Example: ZFT position

Given:

- Circuit with NAND gate 1 and four I/O pads on a 3 x 3 grid
- Pad positions: *In*1 (2,2), *In*2 (0,2), *In*3 (0,0), *Out* (2,0)
- Weighted connections: *c*(*a*,*In*1) = 8, *c*(*a*,*In*2) = 10, *c*(*a*,*In*3) = 2, *c*(*a*,*Out*) = 2

Task: find the ZFT position of cell *a*





4.3.2 Analytic Placement – Force-directed Placement

Example: ZFT position

Given:

- Circuit with NAND gate 1 and four I/O pads on a 3 x 3 grid
- Pad positions: *In*1 (2,2), *In*2 (0,2), *In*3 (0,0), *Out* (2,0)

Solution:

$$x_a^0 = \frac{\sum\limits_{c(i,j)\neq 0} c(a,j)\cdot x_j^0}{\sum\limits_{c(i,j)\neq 0} c(a,j)} = \frac{c(a,In1)\cdot x_{In1} + c(a,In2)\cdot x_{In2} + c(a,In3)\cdot x_{In3} + c(a,Out)\cdot x_{Out}}{c(a,In1)+c(a,In2)+c(a,In3)+c(a,Out)} = \frac{8\cdot 2 + 10\cdot 0 + 2\cdot 0 + 2\cdot 2}{8+10+2+2} = \frac{20}{22} \approx 0.9$$

$$y_a^0 = \frac{\sum\limits_{c(i,j)\neq 0} c(a,j)\cdot y_j^0}{\sum\limits_{c(i,j)\neq 0} c(a,j)} = \frac{c(a,In1)\cdot y_{In1} + c(a,In2)\cdot y_{In2} + c(a,In3)\cdot y_{In3} + c(a,Out)\cdot y_{Out}}{c(a,In1)+c(a,In2)+c(a,In3)+c(a,Out)} = \frac{8\cdot 2 + 10\cdot 2 + 2\cdot 0 + 2\cdot 0}{8+10+2+2} = \frac{36}{22} \approx 1.6$$

ZFT position of cell *a* is (1,2)

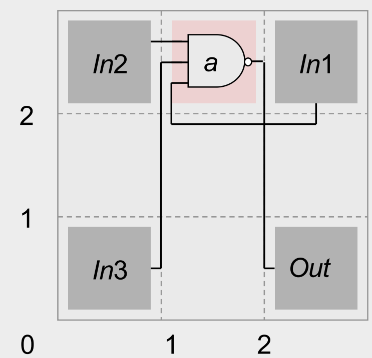
4.3.2 Analytic Placement – Force-directed Placement

Example: ZFT position

Given:

- Circuit with NAND gate 1 and four I/O pads on a 3 x 3 grid
- Pad positions: *In*1 (2,2), *In*2 (0,2), *In*3 (0,0), *Out* (2,0)

Solution:



ZFT position of cell *a* is (1,2)

4.3.2 Analytic Placement – Force-directed Placement

Input: set of all cells *V*
Output: placement *P*

```
P = PLACE(V)                          // arbitrary initial placement
loc = LOCATIONS(P)                    // set coordinates for each cell in P
foreach (cell c ∈ V)
   status[c] = UNMOVED
while (ALL_MOVED(V) || !STOP())       // continue until all cells have been
                                      //   moved or some stopping
                                      //   criterion is reached
   c = MAX_DEGREE(V,status)           // unmoved cell that has largest
                                      //   number of connections
   ZFT_pos = ZFT_POSITION(c)          // ZFT position of c
   if (loc[ZFT_pos] == Ø)             // if position is unoccupied,
      loc[ZFT_pos] = c                //   move c to its ZFT position
   else
      RELOCATE(c,loc)                 // use methods discussed next
   status[c] = MOVED                  // mark c as moved
```

4.3.2 Analytic Placement – Force-directed Placement

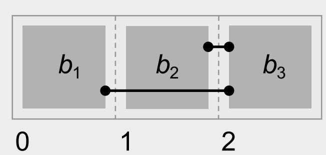
Finding a valid location for a cell with an occupied ZFT position
(*p:* incoming cell, *q*: cell in *p*'s ZFT position)

- If possible, move *p* to a cell position close to *q*.
- Chain move: cell *p* is moved to cells *q*'s location.
  - Cell *q*, in turn, is shifted to the next position. If a cell *r* is occupying this space, cell *r* is shifted to the next position.
  - This continues until all affected cells are placed.
- Compute the cost difference if *p* and *q* were to be swapped.
  If the total cost reduces, i.e., the weighted connection length *L*(*P*) is smaller, then swap *p* and *q*.

4.3.2 Analytic Placement – Force-directed Placement (Example)

Given:

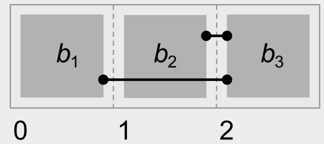
Nets	Weight
$N_1 = (b_1, b_3)$	$c(N_1) = 2$
$N_2 = (b_2, b_3)$	$c(N_2) = 1$

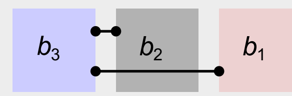


4.3.2 Analytic Placement – Force-directed Placement (Example)

Given:

Nets	Weight
$N_1 = (b_1, b_3)$	$c(N_1) = 2$
$N_2 = (b_2, b_3)$	$c(N_2) = 1$

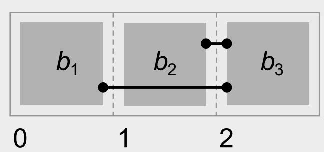


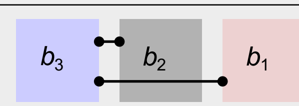
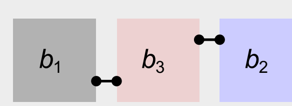
Incoming cell p	ZFT position of cell p	Cell q	$L(P)$ before move	$L(P)$ / placement after move
b_3	$x^0_{b_3} = \dfrac{\sum_{c(b_3,j)\neq 0} c(b_3,j)\cdot x_j^0}{\sum_{c(b_3,j)\neq 0} c(b_3,j)} = \dfrac{2\cdot 0+1\cdot 1}{2+1} \approx 0$	b_1	$L(P) = 5$	$L(P) = 5$  $\Rightarrow$ No swapping of b_3 and b_1

4.3.2 Analytic Placement – Force-directed Placement (Example)

Given:

Nets	Weight
$N_1 = (b_1, b_3)$	$c(N_1) = 2$
$N_2 = (b_2, b_3)$	$c(N_2) = 1$

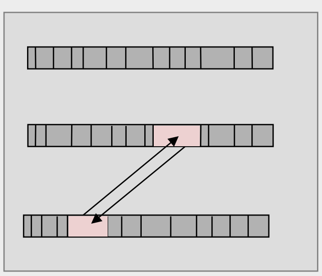


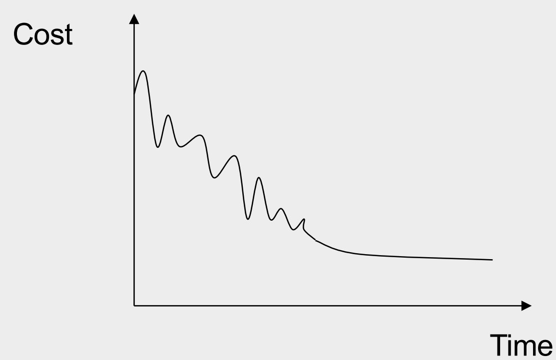
Incoming cell p	ZFT position of cell p	Cell q	$L(P)$ before move	$L(P)$ / placement after move
b_3	$x^0_{b_3} = \dfrac{\sum_{c(b_3,j)\neq 0} c(b_3,j)\cdot x_j^0}{\sum_{c(b_3,j)\neq 0} c(b_3,j)} = \dfrac{2\cdot 0+1\cdot 1}{2+1} \approx 0$	b_1	$L(P) = 5$	$L(P) = 5$  $\rightarrow$ No swapping of b_3 and b_1
b_2	$x^0_{b_2} = \dfrac{\sum_{c(b_2,j)\neq 0} c(b_2,j)\cdot x_j^0}{\sum_{c(b_2,j)\neq 0} c(b_2,j)} = \dfrac{1\cdot 2}{1} = 2$	b_3	$L(P) = 5$	$L(P) = 3$  $\rightarrow$ Swapping of b_2 and b_3

4.3.2 Analytic Placement – Force-directed Placement

- Advantages:
  - Conceptually simple, easy to implement
  - Primarily intended for global placement, but can also be adapted to detailed placement
- Disadvantages:
  - Does not scale to large placement instances
  - Is not very effective in spreading cells in densest regions
  - Poor trade-off between solution quality and runtime
- In practice, FDP is extended by specialized techniques for cell spreading
  - This facilitates scalability and makes FDP competitive

4.3.3 Simulated Annealing





- Analogous to the physical annealing process
  - Melt metal and then slowly cool it
  - Result: energy-minimal crystal structure
- Modification of an initial configuration (placement) by moving/exchanging of randomly selected cells
  - Accept the new placement if it improves the objective function
  - If no improvement: Move/exchange is accepted with temperature-dependent (i.e., decreasing) probability

4.3.3 Simulated Annealing – Algorithm

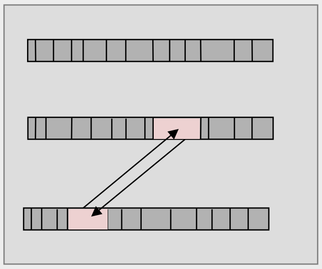
Input: set of all cells V
Output: placement P

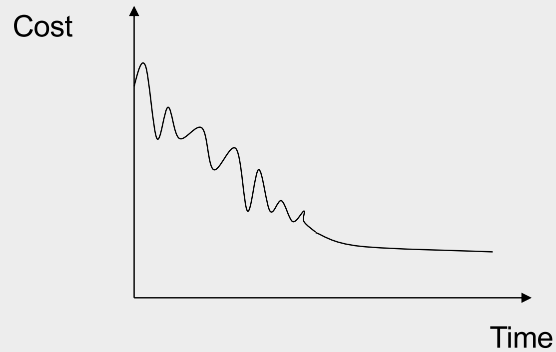
```
T = T_0                                   // set initial temperature
P = PLACE(V)                              // arbitrary initial placement
while (T > T_min)
   while (!STOP())                        // not yet in equilibrium at T
      new_P = PERTURB(P)
      Δcost = COST(new_P) – COST(P)
      if (Δcost < 0)                      // cost improvement
         P = new_P                        // accept new placement
      else                                // no cost improvement
         r = RANDOM(0,1)                  // random number [0,1)
         if (r < e^(-Δcost/T))            // probabilistically accept
           P = new_P
   T = α · T                              // reduce T, 0 < α < 1
```


4.3.3 Simulated Annealing

- Advantages:
  - Can find global optimum (given sufficient time)
  - Well-suited for detailed placement
- Disadvantages:
  - Very slow
  - To achieve high-quality implementation, laborious parameter tuning is necessary
  - Randomized, chaotic algorithms - small changes in the input lead to large changes in the output
- Practical applications of SA:
  - Very small placement instances with complicated constraints
  - Detailed placement, where SA can be applied in small windows (not common anymore)
  - FPGA layout, where complicated constraints are becoming a norm

4.3.3 Simulated Annealing





- Analogous to the physical annealing process
  - Melt metal and then slowly cool it
  - Result: energy-minimal crystal structure
- Modification of an initial configuration (placement) by moving/exchanging of randomly selected cells
  - Accept the new placement if it improves the objective function
  - If no improvement: Move/exchange is accepted with temperature-dependent (i.e., decreasing) probability

4.3.4 Modern Placement Algorithms

- Predominantly analytic algorithms
- Solve two challenges: interconnect minimization and cell overlap removal (spreading)
- Two families:

Quadratic placers	Non-convex optimization placers

4.3.4 Modern Placement Algorithms

Quadratic placers

Non-convex optimization placers

- Solve large, sparse systems of linear equations (formulated using force-directed placement) by the Conjugate Gradient algorithm
- Perform cell spreading by adding fake nets that pull cells away from dense regions toward carefully placed anchors

4.3.4 Modern Placement Algorithms

Quadratic placers

Non-convex optimization placers

- Model interconnect by sophisticated differentiable functions, e.g., log-sum-exp is the popular choice
- Model cell overlap and fixed obstacles by additional (non-convex) functional terms
- Optimize interconnect by the non-linear Conjugate Gradient algorithm
- Sophisticated, slow algorithms
- All leading placers in this category use netlist clustering to improve computational scalability (this further complicates the implementation)

4.3.4 Modern Placement Algorithms

Quadratic Placement	Non-convex optimization placers

Pros and cons:

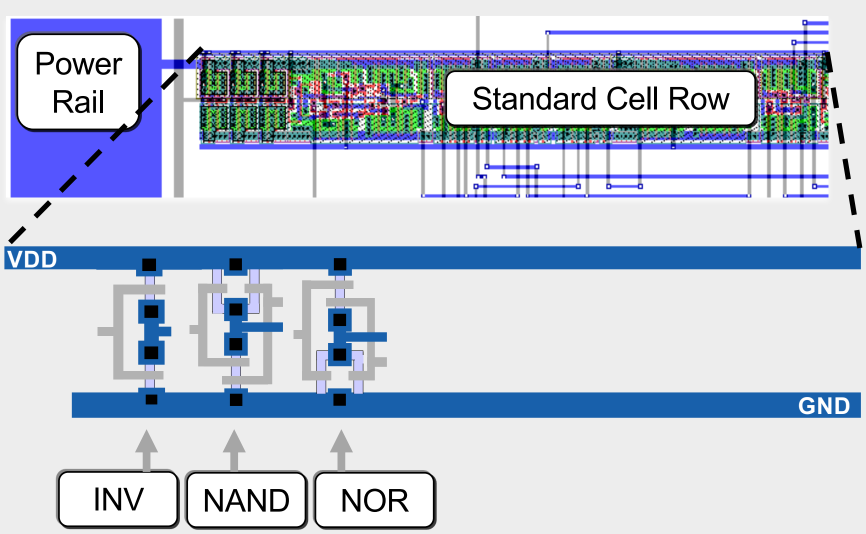
- Quadratic placers are simpler and faster, easier to parallelize
- Non-convex optimizers tend to produce better solutions
- As of 2011, quadratic placers are catching up in solution quality while running 5-6 times faster [1]

[1] M.-C.Kim, D. Lee, I. L. Markov: SimPL: An effective placement algorithm. ICCAD 2010: 649-656

4.4 Legalization and Detailed Placement

- Global placement must be legalized
  - Cell locations typically do not align with power rails
  - Small cell overlaps due to incremental changes, such as cell resizing or buffer insertion
- Legalization seeks to find legal, non-overlapping placements for all placeable modules
- Legalization can be improved by detailed placement techniques, such as
  - Swapping neighboring cells to reduce wirelength
  - Sliding cells to unused space
- Software implementations of legalization and detailed placement are often bundled

Legal positions of standard cells between VDD and GND rails



Summary of Chapter 4 – Problem Formulation and Objectives

- Row-based standard-cell placement
  - Cell heights are typically fixed, to fit in rows (but some cells may have double and quadruple heights)
  - Legal cell sites facilitate the alignment of routing tracks, connection to power and ground rails
- Wirelength as a key metric of interconnect
  - Bounding box half-perimeter (HPWL)
  - Cliques and stars
  - RMSTs and RSMTs
- Objectives: wirelength, routing congestion, circuit delay
  - Algorithm development is usually driven by wirelength
  - The basic framework is implemented, evaluated and made competitive on standard benchmarks
  - Additional objectives are added to an operational framework

Summary of Chapter 4 – Global Placement

- Combinatorial optimization techniques: min-cut and simulated annealing
  - Can perform both global and detailed placement
  - Reasonably good at small to medium scales
  - SA is very slow, but can handle a greater variety of constraints
  - Randomized and chaotic algorithms – small changes at the input can lead to large changes at the output
- Analytic techniques: force-directed placement and non-convex optimization
  - Primarily used for global placement
  - Unrivaled for large netlists in speed and solution quality
  - Capture the placement problem by mathematical optimization
  - Use efficient numerical analysis algorithms
  - Ensure stability: small changes at the input can cause only small changes at the output
  - Example: a modern, competitive analytic global placer takes 20mins for global placement of a netlist with 2.1M cells (single thread, 3.2GHz Intel CPU) [1]

[1] M.-C.Kim, D. Lee, I. L. Markov: SimPL: An effective placement algorithm. ICCAD 2010: 649-656

Summary of Chapter 4 – Legalization and Detailed Placement

- Legalization ensures that design rules & constraints are satisfied
  - All cells are in rows
  - Cells align with routing tracks
  - Cells connect to power & ground rails
  - Additional constraints are often considered, e.g., maximum cell density
- Detailed placement reduces interconnect, while preserving legality
  - Swapping neighboring cells, rotating groups of three
  - Optimal branch-and-bound on small groups of cells
  - Sliding cells along their rows
  - Other local changes
- Extensions to optimize routed wirelength, routing congestion and circuit timing
- Relatively straightforward algorithms, but high-quality, fast implementation is important
- Most relevant after analytic global placement, but are also used after min-cut placement
- Rule of thumb: 50% runtime is spent in global placement, 50% in detailed placement [1]

[1] M.-C.Kim, D. Lee, I. L. Markov: SimPL: An effective placement algorithm. ICCAD 2010: 649-656